Fair and Efficient Social Decision-Making

CSCI 699

Voting: Axioms I

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Credit for the slides and most of visuals: Nisarg Shah and Ariel D. Procaccia

Voting Theory

Social Choice Theory

- Mathematical theory for aggregating individual preferences into collective decisions

Voting Theory

- Originated in ancient Greece
- Formal foundations
  - 13th Century (Ramon Llull)
  - 18th Century (Marquis de Condorcet and Jean-Charles de Borda)
  - 19th Century: Charles Dodgson (a.k.a. Lewis Carroll)
  - 20th Century: Nobel prizes to Kenneth Arrow and Amartya Sen

Voting Theory

- We want to select a collective decision based on (possibly different) individual preferences
  - Presidential election, restaurant/movie selection for group activity, committee selection, facility location, …
- Resource allocation is a special case
  - You can think of all possible allocations as the different "outcomes"
    - A very restricted case due to lots of ties
    - An agent is indifferent among all allocations in which the resources *she* gets are the same
  - We want to study the general case

Voting Framework

- Set of voters $N = \{1, \dots, n\}$
- Set of alternatives A, $|A| = m$
- Voter i has a preference ranking $\succ_i$ over the alternatives
- Preference profile $\overrightarrow{\succ}$ is the collection of all voters' rankings
- Other Ballot Types:
  - Approval Votes
  - Scores/Stars

1	2	3
a	c	b
b	a	a
c	b	c

Voting Framework

- Social choice function f
  - Takes as input a preference profile $\vec{\succ}$
  - Returns an alternative $a \in A$
- Social welfare function f
  - Takes as input a preference profile $\vec{\succ}$
  - Returns a societal preference $\succ^*$
- For now, voting rule = social choice function

1	2	3
a	c	b
b	a	a
c	b	c

Voting Rules

- Plurality
  - Each voter awards one point to her top alternative
  - Alternative with the most point wins
  - Most frequently used voting rule
  - Almost all political elections use plurality

1	2	3	4	5
a	a	a	d	e
b	b	b	c	c
c	c	c	a	d
d	d	d	e	b
e	e	e	b	a

Winner
a

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  - Almost all political elections use plurality

1	2	3	4	5
a	a	c	d	e
b	b	b	c	c
c	c	a	a	d
d	d	d	e	b
e	e	e	b	a

Winner
a

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- Plurality
  - Each voter awards one point to her top alternative
  - Alternative with the most point wins
  - Most frequently used voting rule
  - Almost all political elections use plurality

- Problem?

1	2	3	4	5
a	a	a	b	b
b	b	b	c	c
c	c	c	d	d
d	d	d	e	e
e	e	e	a	a

Winner
a

Voting Rules

- Borda Count
  - Each voter awards $m - k$ points to alternative at rank k
  - Alternative with the most points wins
  - Proposed by Ramon Llull in the 13th Century but named after 18th Century work by Jean-Charles de Borda
  - Used for elections to the national assembly of Slovenia

1	2	3
a (2)	c (2)	b (2)
b (1)	a (1)	a (1)
c (0)	b (0)	c (0)

Total
a: 2+1+1 = 4
b: 1+0+2 = 3
c: 0+2+0 = 2

Winner
a

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1	2	3	4	5
a	a	a	b	b
b	b	b	c	c
c	c	c	d	d
d	d	d	e	e
e	e	e	a	a

Total
a: 4+4+4+0+0 = 12
b: 3+3+3+4+4 = 13
….

Winner
b

Current uses [edit]

Political uses [edit]

The Borda count is used for certain political elections in at least three countries, Slovenia and the tiny Micronesian nations of Kiribati and Nauru. In Slovenia, the Borda count is used to elect two of the ninety members of the National Assembly: one member represents a constituency of ethnic Italians, the other a constituency of the Hungarian minority. As noted above, members of the Parliament of Nauru are elected based on a variant of the Borda count that involves two departures from the normal practice: (1) multi-seat constituencies, of either two or four seats, and (2) a point-allocation formula that involves increasingly small fractions of points for each ranking, rather than whole points. In Kiribati, the president (or *Beretitenti*) is elected by the plurality system, but a variant of the Borda count is used to select either three or four candidates to stand in the election. The constituency consists of members of the legislature (*Maneaba*). Voters in the legislature rank only four candidates, with all other candidates receiving zero points. Since at least 1991, tactical voting has been an important feature of the nominating process.

The Republic of Nauru became independent from Australia in 1968. Before independence, and for three years afterwards, Nauru used instant-runoff voting, importing the system from Australia, but since 1971, a variant of the Borda count has been used.

The modified Borda count has been used by the Green Party of Ireland to elect its chairperson.[6][7]

The Borda count has been used for non-governmental purposes at certain peace conferences in Northern Ireland, where it has been used to help achieve consensus between participants including members of Sinn Féin, the Ulster Unionists, and the political wing of the UDA.

Other uses [edit]

The Borda count is used in elections by some educational institutions in the United States.

- University of Michigan
  - Central Student Government
  - Student Government of the College of Literature, Science and the Arts (LSASG)
- University of Missouri: officers of the Graduate-Professional Council
- University of California Los Angeles: officers of the Graduate Student Association
- Harvard University: officers of the Civil Liberties Union
- Southern Illinois University at Carbondale: officers of the Faculty Senate,
- Arizona State University: officers of the Department of Mathematics and Statistics assembly.
- Wheaton College, Massachusetts: faculty members of committees.
- College of William and Mary: members of the faculty personnel committee of the School of Business Administration (tie-breaker).

Borda count in real life

The Borda count is used in elections by some professional and technical societies.

- International Society for Cryobiology: Board of Governors.
- Tempo sustainable design network: management committee.
- U.S. Wheat and Barley Scab Initiative: members of Research Area Committees.
- X.Org Foundation: Board of Directors.

The OpenGL Architecture Review Board uses the Borda count as one of the feature-selection methods.

The Borda count is used to determine winners for Toastmasters International speech contests. Judges offer a ranking of their top three speakers, awarding them three points, two points, and one point, respectively. All unranked candidates receive zero points.

The modified Borda count is used to elect the President for the United States member committee of AIESEC.

The Borda count, and points-based systems similar to it, are often used to determine awards in competitions.

The Borda count is a popular method for granting sports awards in the United States. Uses include:

- MLB Most Valuable Player Award (baseball)
- Heisman Trophy (college football)[8]
- Ranking of NCAA college teams

The Eurovision Song Contest uses a positional voting method similar to the Borda count, with a different distribution of points: only the top ten entries are considered in each ballot, the favorite entry receiving 12 points, the second-placed entry receiving 10 points, and the other eight entries getting points from 8 to 1. Although designed to favor a clear winner, it has produced very close races and even a tie.

The People's Remix Competition uses a Borda variant where each voter ranks only the top three contestants.

The Borda count is used for wine trophy judging by the Australian Society of Viticulture and Oenology, and by the RoboCup autonomous robot soccer competition at the Center for Computing Technologies, in the University of Bremen in Germany.

The Finnish Associations Act lists three different modifications of the Borda count for holding a proportional election. All the modifications use fractions, as in Nauru. A Finnish association may choose to use other methods of election, as well.[9]

Voting Rules

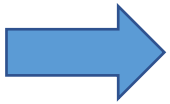
- Positional Scoring Rules
  - Defined by a score vector $\vec{s} = (s_1, \dots, s_m)$
  - Each voter gives s_k points to alternative at rank k

- A family containing many important rules
  - Plurality = $(1,0, \dots, 0)$
  - Borda = $(m-1, m-2, \dots, 0)$
  - k-approval = $(1, \dots, 1,0, \dots, 0)$ ← top k get 1 point each
  - Veto = $(0, \dots, 0, -1)$
  - …

Voting Rules

- Plurality with runoff
  - **First round:** two alternatives with the highest plurality scores survive
  - **Second round:** between these two alternatives, select the one that majority of voters prefer
- Similar to the French presidential election system
  - Problem: vote division
  - Happened in the 2002 French presidential election

1	2	3	4	5
a	a	c	c	e
b	b	b	d	c
c	c	a	a	d
d	d	d	e	b
e	e	e	b	a



1	2	3	4	5
a	a	c	c	c
c	c	a	a	a

Winner
c

Voting Rules

- Single Transferable Vote (STV)
  - $m - 1$ rounds
  - In each round, the alternative with the least plurality votes is eliminated
  - Alternative left standing is the winner
  - Used in Ireland, Malta, Australia, New Zealand, …

- STV has been strongly advocated for due to various reasons

STV Example

2 voters	2 voters	1 voter
a	b	c
b	a	d
c	d	b
d	c	a

2 voters	2 voters	1 voter
a	b	c
b	a	b
c	c	a

2 voters	2 voters	1 voter
a	b	b
b	a	a

2 voters	2 voters	1 voter
b	b	b

Voting Rules

- Kemeny's Rule
  - Social welfare function (selects a ranking)
  - Let $n_{a \succ b}$ be the number of voters who prefer a to b
  - Select a ranking σ of alternatives = for every pair (a, b) where $a \succ_\sigma b$, we make $n_{b \succ a}$ voters unhappy
  - Total unhappiness $K(\sigma) = \sum_{(a,b): a \succ_\sigma b} n_{b \succ a}$
  - Select the ranking σ^* with minimum total unhappiness
- Social choice function
  - Choose the top alternative in the Kemeny ranking

Kemeny Example

2 voters	2 voters	1 voter
a	b	c
b	a	d
c	d	b
d	c	a

- $K(a \succ b \succ c \succ d)$
  - 0 x first 2 voters
  - 2 x next 2 voters
  - 5 x last voter
  - 9 in total

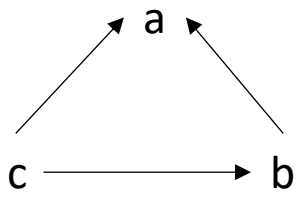
- $K(b \succ a \succ c \succ d)$
  - 1 x first 2 voters
  - 1 x next 2 voters
  - 4 x last voter
  - 8 in total

Condorcet Winner

- Definition
  - Alternative x defeats y in a pairwise election if a *strict* majority of voters prefer x to y
  - Alternative x is a Condorcet winner if it defeats every other alternative in a pairwise election
- Question
  - Can there be two Condorcet winners?

1	2	3
a	b	c
b	a	a
c	c	b

Majority Preference

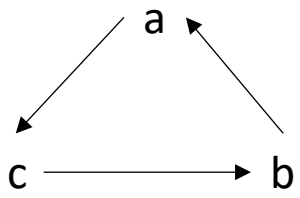


Condorcet Winner

- Definition
  - Alternative x defeats y in a pairwise election if a *strict* majority of voters prefer x to y
  - Alternative x is a Condorcet winner if it defeats every other alternative in a pairwise election
- Condorcet paradox
  - No Condorcet winner when the majority preference is cyclic

1	2	3
a	b	c
b	c	a
c	a	b

Majority Preference



Condorcet Consistency

- Condorcet consistency
  - A voting rule is Condorcet consistent if it selects the Condorcet winner whenever one exists
  - On preference profiles where there is no Condorcet winner, it is free to output any winner

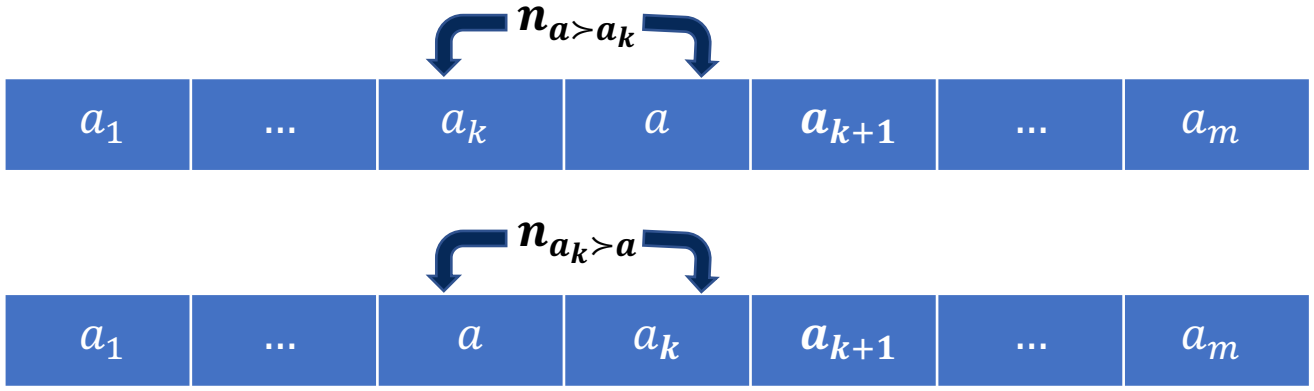
- Question: Which rule is Condorcet consistent:

  1. Plurality
  2. Borda
  3. Both rules
  4. Neither one

- ***NOT* Condorcet consistent:** all positional scoring rules (plurality, Borda, …), plurality with runoff, STV

Condorcet Consistency

- Condorcet consistency
  - A voting rule is Condorcet consistent if it selects the Condorcet winner whenever one exists
  - On preference profiles where there is no Condorcet winner, it is free to output any winner
- Question: Is Kemeny rule Condorcet consistent?
  - Yes! Let a be the Condorcet winner



Condorcet Consistency

- Copeland
  - Score(x) = # alternatives x beats in pairwise elections
  - Select x^* with the maximum score
  - Condorcet consistent (Why?)

Condorcet Consistency

- Maximin
  - Score(x) = $\min_y n_{x \succ y}$
  - Select x^* with the maximum score
  - Also Condorcet consistent (Why?)

1	2	3	4	5
a	a	a	d	e
b	b	b	c	c
c	c	c	a	d
d	d	d	e	b
e	e	e	b	a

e.g. $Score(a) = \min\{n_{a \succ b}, n_{a \succ c}, n_{a \succ d}, n_{a \succ e}\} = \min\{4,3,3,4\}$

Majority Consistency

- Majority consistency
  - If a strict majority of voters rank alternative x first, then x must be the winner.

- Question: What is the relation between majority consistency and Condorcet consistency?

  1. Majority consistency ⇒ Condorcet consistency
  2. Condorcet consistency ⇒ Majority consistency
  3. Equivalent
  4. Incomparable

- Question: Which rule is Majority consistent:

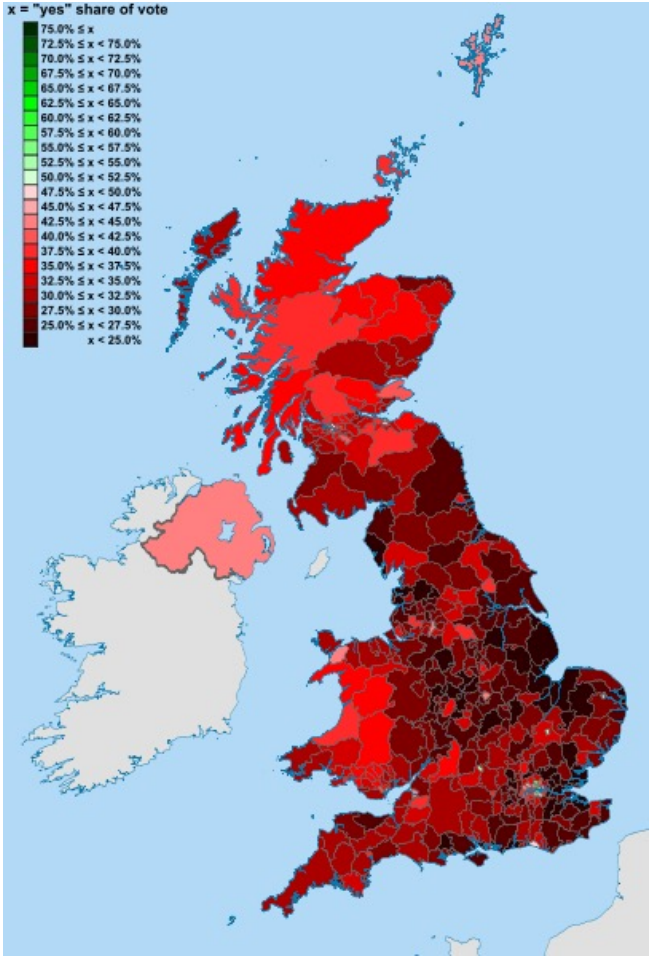
  1. Plurality
  2. Borda
  3. Both rules
  4. Neither one

Which rule to use?

- We just introduced infinitely many rules
  - (Recall positional scoring rules…)
- How do we know which is the “right” rule to use?
  - Various approaches
  - Axiomatic, statistical, utilitarian, …
- How do we ensure good incentives without using money?
  - Bad luck!

Is Social Choice Practical?

- UK referendum: Choose between plurality and STV for electing MPs
- Academics agreed STV is better...
- ...but STV seen as beneficial to a particular politician
- Hard to change political elections!



Incentives

- Can a voting rule incentivize voters to truthfully report their preferences?

- Strategyproofness
  - A voting rule is strategyproof if a voter cannot submit a false preference and get a more preferred alternative (under her true preference) elected, irrespective of the preferences of other voters
  - Formally, a voting rule f is strategyproof if for every preference profile $\overrightarrow{\succ}$, voter i, and preference $\succ_i'$, we have

$$f\left(\overrightarrow{\succ}\right) \succcurlyeq_i f\left(\overrightarrow{\succ}_{-i}, \succ_i'\right)$$

  - Question: What is the relation between $f\left(\overrightarrow{\succ}\right)$ and $f\left(\overrightarrow{\succ}_{-i}, \succ_i'\right)$ according to $\succcurlyeq_i'$?

Strategyproofness

- Question: Is Borda Count strategyproof?
- Example:
  - In the true profile, b wins
  - Voter 3 can make a win by pushing b to the end

Winner
b

1	2	3
b	b	a
a	a	b
c	c	c
d	d	d

1	2	3
b	b	a
a	a	c
c	c	d
d	d	b

Winner
a

Borda's Response to Critics

My scheme is intended only for honest men!

Random 18th century French dude

Strategyproofness

- None of the rules we saw are strategyproof!

Dictatorship

- Are there any strategyproof rules?
  - Sure
- Dictatorial voting rule
  - The winner is always the most preferred alternative of voter i
- Constant voting rule
  - The winner is always the same

Constant function

- Not satisfactory (for most cases)