



**Fair and**



**Efficient**



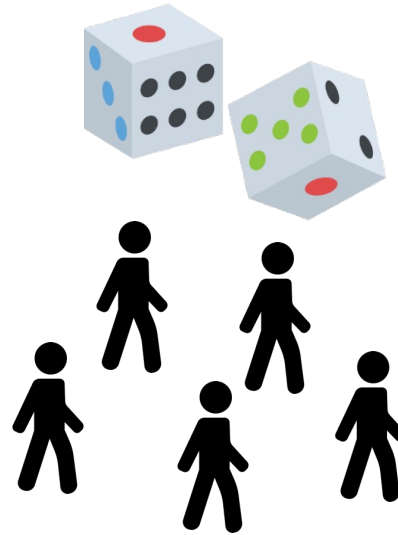
**Social Decision-Making**

CSCI 699

**Fair Division 3: Sortition**

**Evi Micha**

# Citizens' Assemblies



# Citizens' Assemblies

## Athens

4<sup>th</sup> century BC

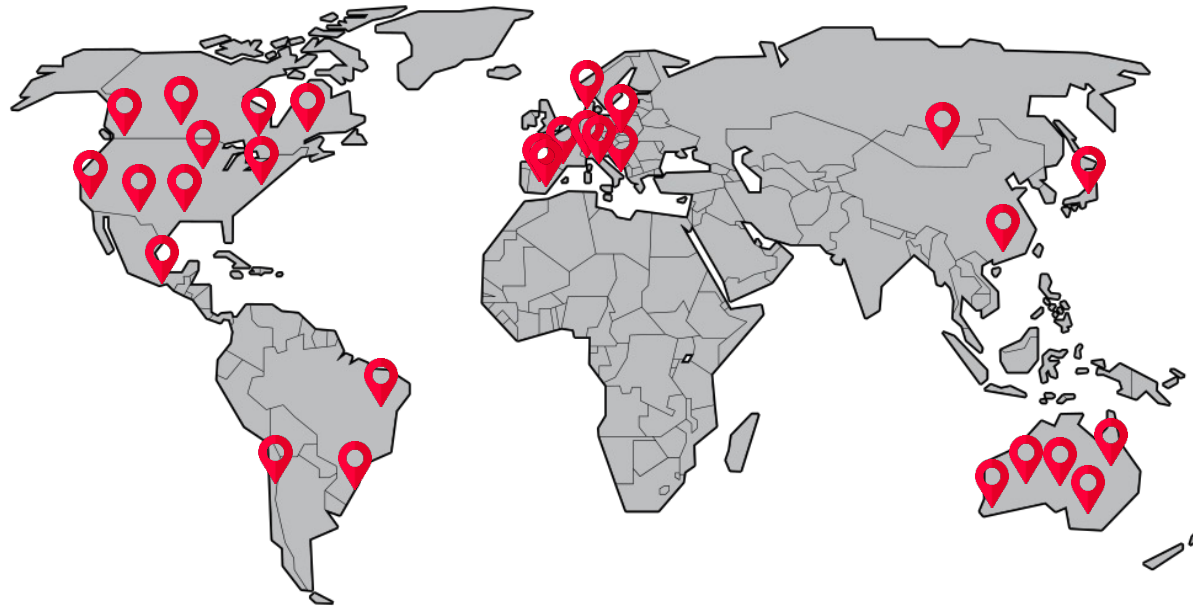
## Florence

14<sup>th</sup> and 15<sup>th</sup> century

## Lombardy and Venice

12<sup>th</sup> to 18<sup>th</sup> century

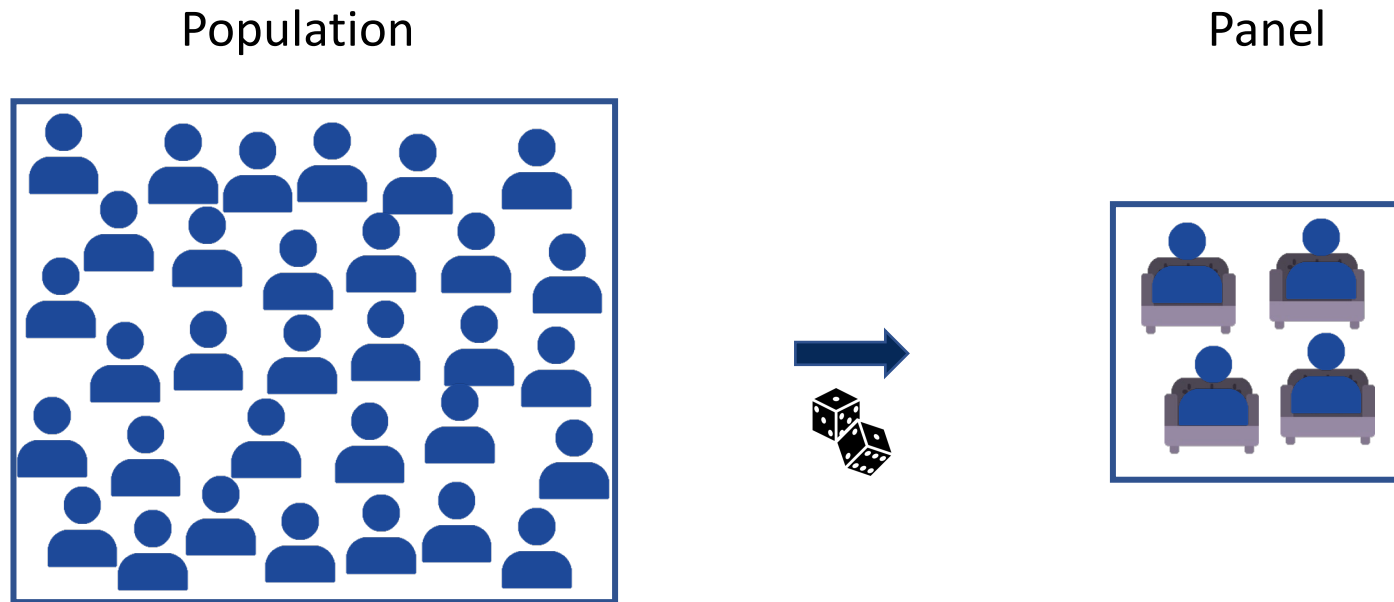
# Citizens' Assemblies



Belgium

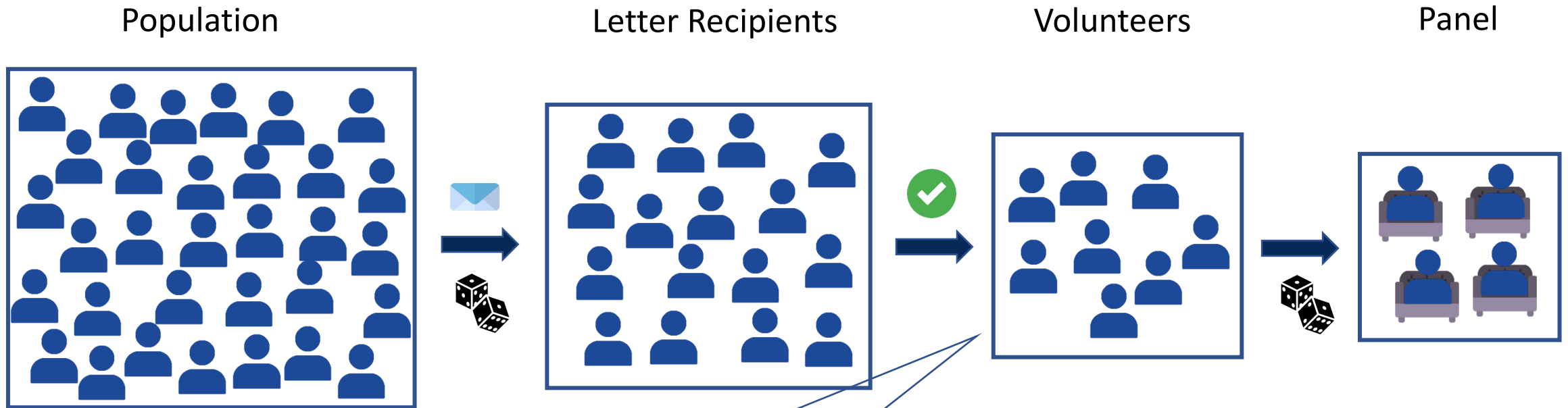
- Permanent Citizens Assemblies at a national level
- The recommendations *should* be followed up within nine months

# Ideal Sortition Execution



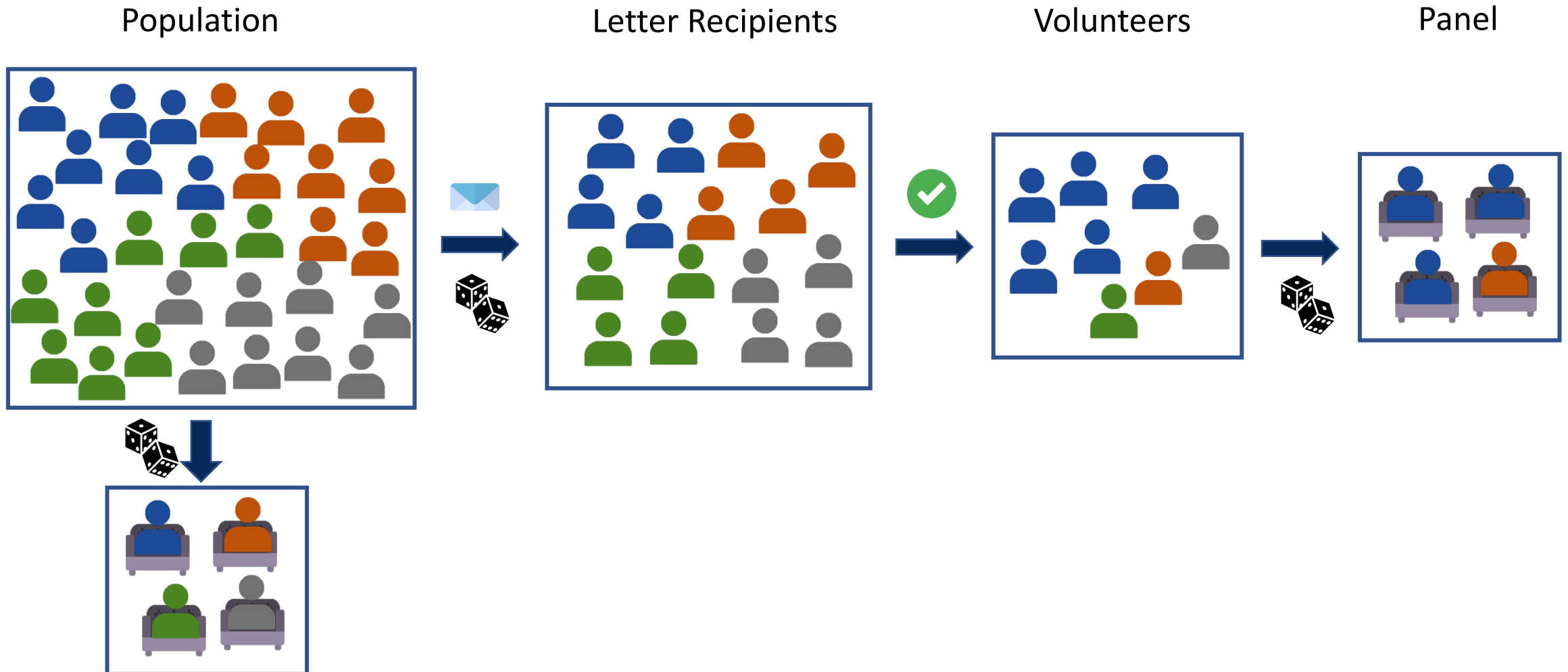
- ✓ **Fairness:** Everyone has equal chance of being on the panel
- ✓ **Proportional Representation:** A panel selected uniformly at random reflects the composition of the population, *in expectation*

# Sortition in Practice



**Climate Assembly UK 2020**  
Only 1,727 citizens accepted to participate out of the 30,000 recipients of the invitation!

# Sortition in Practice



# Features Quotas

## ➤ Gender

- At least 47% of the representatives should females
- At least 47% of the representatives should males
- At least 3% of the representatives should self-identify non-binary
- 

## ➤ Age

- At least 30% of the representatives  $\leq 35$
- At least 25% of the representatives  $\geq 65$

## ➤ Geography

- At least 35% of the representatives form the south
- At least 30% of the representatives form the rural regions

## ➤ ...

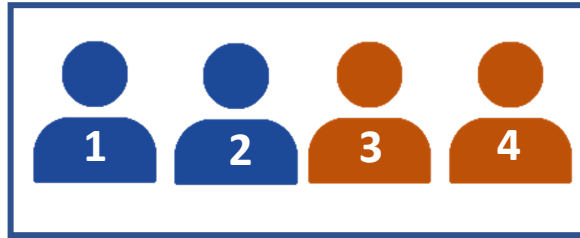


# Model

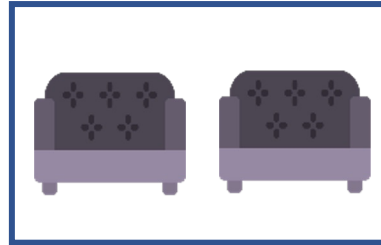
- A set of **volunteers**  $N = \{1, 2, \dots, n\}$
- A set of **features**  $F$ , where each  $f \in F$  gets values in  $V_f$ 
  - E.g. feature gender  $g$  gets values in  $V_g = \{non - binary, female, male\}$
- Each volunteer  $i$  is characterized by the features
- A set of **quotas** such that for each  $f \in F$  and  $v_f \in V_f$ , the number of representatives with feature  $f$  equal to  $v_f$  should be at least  $\ell_{v_f}$  and at most  $u_{v_f}$
- **Goal**: Find a lottery over panels of size  $k$  such that
  - Fairness: Each individual is selected with probability  $n/k$  (**ex-ante requirement**)
  - Representation: Each panel in the lottery satisfies the quotas (**ex-post requirement**)

# Fairness and Representation

Volunteers



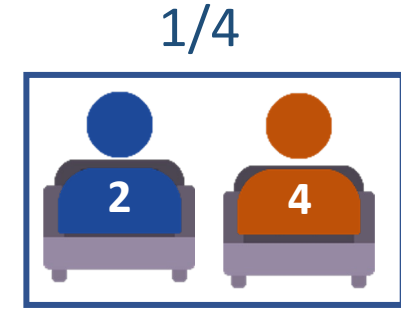
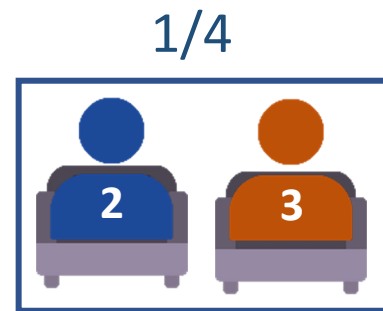
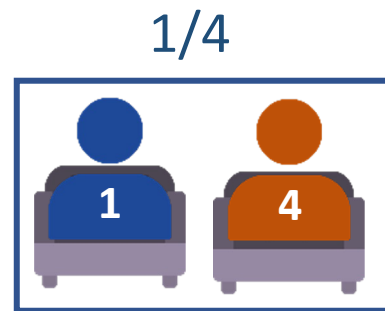
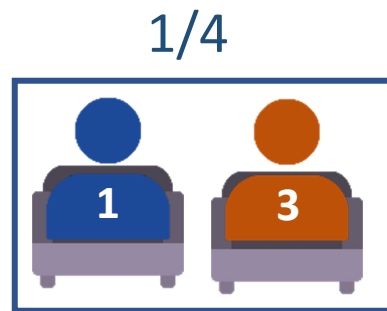
Panel



Quotas

- One representative should be blue
- One representative should be orange

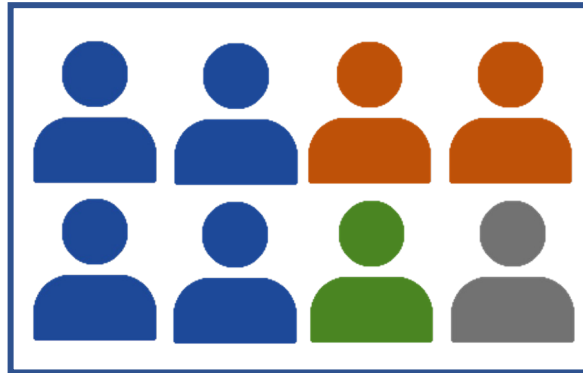
Fair and Representative Lottery



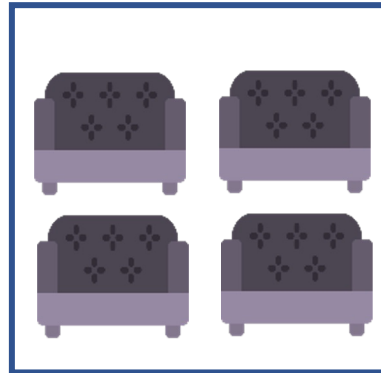
- **Question:** Can we always ensure fairness and representation?

# Fairness and Representation

Volunteers



Panel



Quotas

- One representative should be blue
- One representative should be orange
- One representative should be green
- One representative should be grey

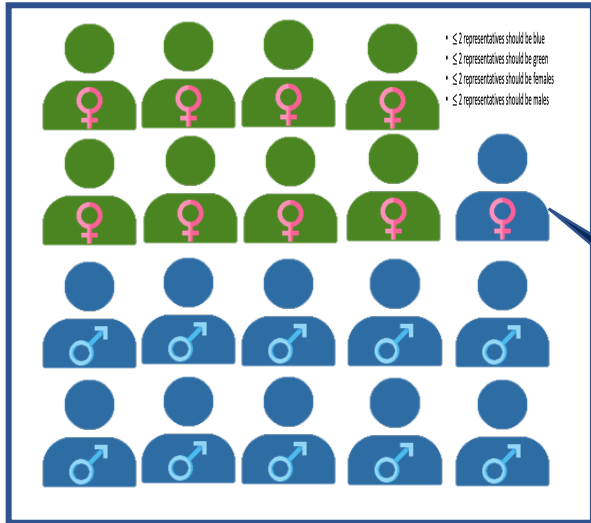
- **Theorem [FGGHP, 2021]:** For a given set of agents, panel size, and set of features with associated quotas, it is NP-hard to decide whether there exists a panel that satisfies the quotas

# A Greedy Algorithm

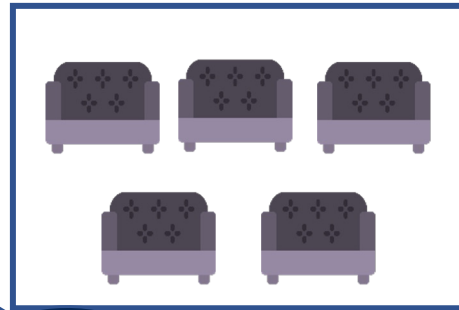
- Start with an empty panel
- At each iteration for each  $f \in F$  and  $v_f \in V_f$  measure the need for  $v_f$  as following:
  - $need_{v_f} = \frac{\ell_{v_f} - (\# \text{ panel members with feature } f \text{ equal to } v_f)}{\# \text{ remaining pool members with feature } f \text{ equal to } v_f}$
- Choose  $v_f$  with the highest need
- Choose a representative at random among all the individuals in the pool with feature  $f$  equal to  $v_f$
- If for some  $v_f$ , there are  $u_{v_f}$  members in the panel with feature  $f$  equal to  $v_f$ , then remove from the pool all the individuals with feature  $f$  equal to  $v_f$
- If the final panel does not satisfy the quotas, restart

# A Greedy Algorithm

Volunteers



Panel



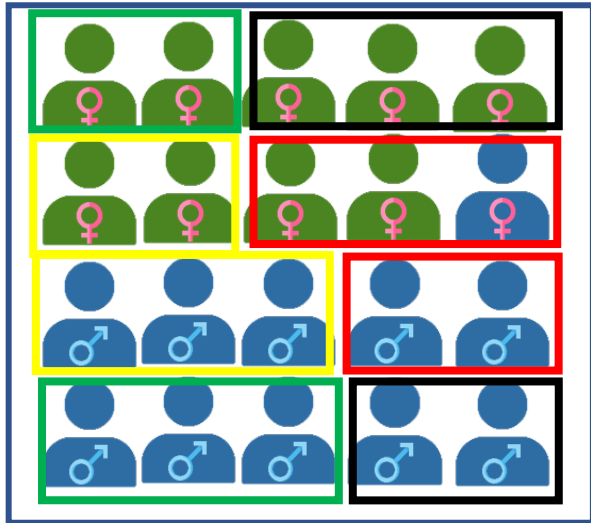
Quotas

- $\leq 2$  representatives should be blue
- $\leq 2$  representatives should be green
- $\leq 2$  representatives should be females
- $\leq 2$  representatives should be males

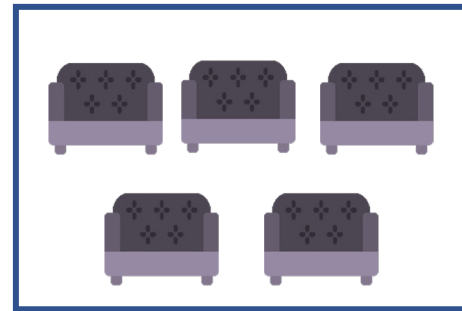
|         | Need of Green | Need of Blue | Need of Female | Need of Male |
|---------|---------------|--------------|----------------|--------------|
| Round 1 | 2/9           | 2/11         | 2/10           | 2/10         |
| Round 2 | 1/8           | 2/11         | 1/9            | 2/10         |
| Round 3 | 1/8           | 1/10         | 1/9            | 1/9          |
| Round 4 | 0/7           | 1/10         | 0/9            | 1/9          |
| Round 5 | 0/7           | 0/9          | 0/9            | 0/8          |

# Fair Lottery

Volunteers



Panel



Quotas

- $\leq 2$  representatives should be blue
- $\leq 2$  representatives should be green
- $\leq 2$  representatives should be females
- $\leq 2$  representatives should be males

# Fair Lottery

- **Maximum Nash Social Welfare**
  - Return a lottery over quota-feasible panels of size  $k$  such that  $\max \prod_{i \in N} \Pr[i \text{ is selected}]$
- **Leximin**
  - Return a lottery over quota-feasible panels of size  $k$  such that  $\max \min_{i \in N} \Pr[i \text{ is selected}]$  subject to that maximize the second minimum probability, etc.
- **Theorem [FGGHP, 2021]: Maximum Nash Social Welfare and Leximin** equalize volunteers' selection probabilities whenever the quotas make it feasible.
- **Question:** What about maximizing the utilitarian social welfare, i.e. sum of probabilities?

# Axiomatic Properties

- **Committee Monotonicity:** When  $k$  increases, the selection probability of all volunteers weakly increases
- **Theorem [FGGHP, 2021]:** No selection algorithm can guarantee committee monotonicity
- **Proof:**

|   | ★ | ❤️ |
|---|---|----|
| 👤 | 1 | 0  |
| 👤 | 0 | 1  |
| 👤 | 1 | 1  |

$$1 \times \star = 1 \quad 1 \times \heartsuit = 1$$

- When  $k=1$ , {👤}
- When  $k=2$ , {👤, 👤}



# Axiomatic Properties

- **Population Monotonicity:** When more volunteers are added, the selection probability of all the existing volunteers weakly decreases
- **Theorem [FGGHP, 2021]:** No selection algorithm can guarantee committee monotonicity